Detecting Carbon Nanotube Orientation with Topological Data Analysis of SEM Images

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Carbon Nanotube Material

High-performance carbon nanotube (CNT) materials are in high demand as a result of their extraordinary mechanical, electrical and thermal properties.

CNT orientation alignment is an important property in the fabrication of ultra-strong CNT composites.

It is fundamentally important to evaluate and quantify the degree of alignment using various characterization methods.

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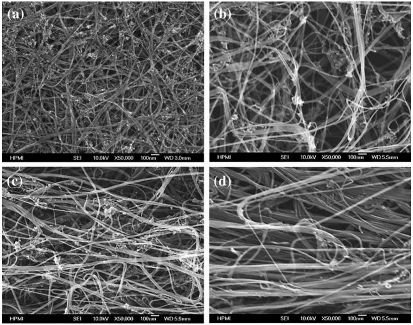
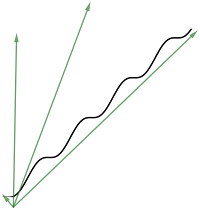
What does orientation alignment mean?

Following figure shows SEM (scanning electron microscope) images of some sample CNT materials.



Goal: To develop effective ways of quantifying the degree of alignment using **topological data analysis**.

Intuition



The length of each vector fits our intuition about how well the tube is oriented in that direction.

Based on this easy observation we use **persistent homology** to produce similar measurements, even though in which the nanotubes may intersect or overlap with each other.

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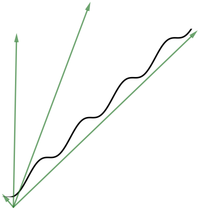

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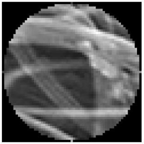
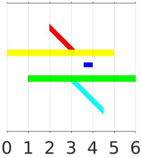
Pre-processing

We apply a Canny edge detector whose output is a binary image, as illustrated in Fig. 8 (ii). Fig. 8 (iii) is a "cartoon" of a nanotube array.



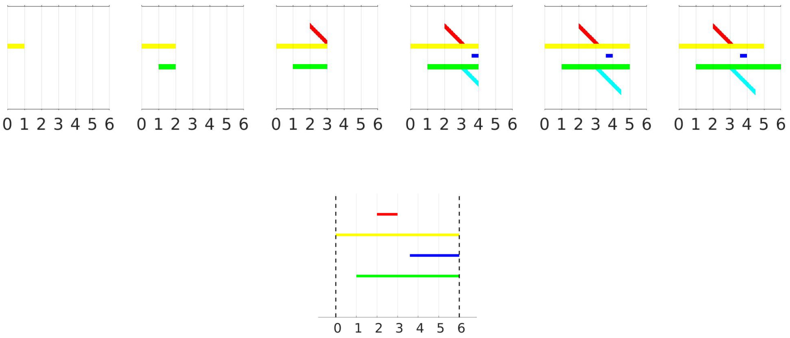
(i)

(ii)

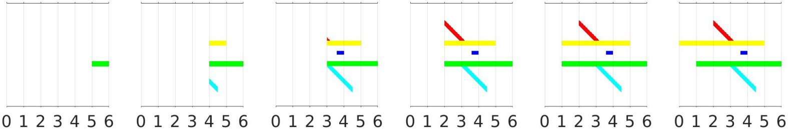


(iii)

An explanation of Persistent Homology Barcode



Roughly speaking, a bar in a barcode records information such as when a new component appears, when it merges into another component while the space grows.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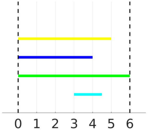
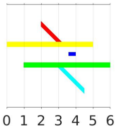
0 1 2 3 4 5 6
0 1 2 3 4 5 6
0 1 2 3 4 5 6
0 1 2 3 4 5 6
0 1 2 3 4 5 6


0 1 2 3 4 5 6

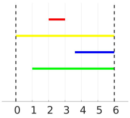
Variation Function

In the following tabular V represents horizontal extension of each branch of X; L_a represents length of each bar in barcode (a); L_b represents length of each bar in barcode (b).

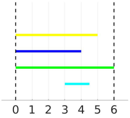
Branch	V	L_a	L_b	$L_a + L_b - V$
Red	1	1	0	0
Yellow	5	6	5	6
Blue	0.5	2.5	4	6
Orange	5	5	6	6
Cyan	1.5	0	1.5	0



X



(a)



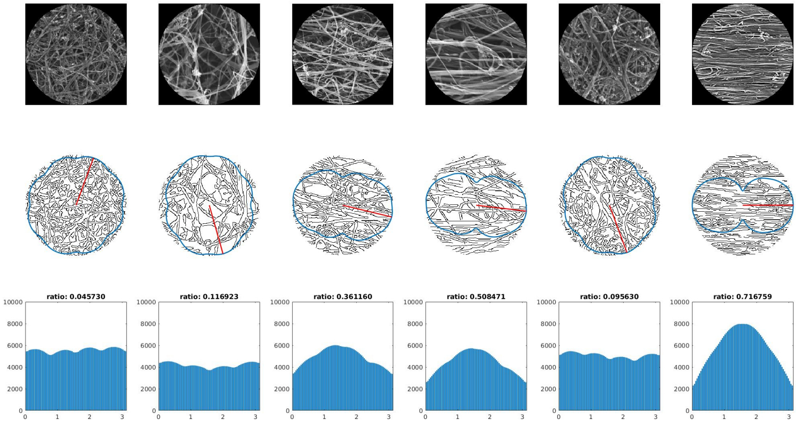
(b)

Fomulation

1. $L_a + L_b - V$ always equal to 6 or 0;
2. $L_a + L_b - V = 6$ iff. the branch does not merge in both filtrations;
3. 6 equals the total horizontal scan range of those filtrations;
4. The number of branches for which $L_a + L_b - V$ exceeds 6 equal $b_0(X)$.

$$V(X, \theta) := \sum_{i=1}^{m} \ell(I_i) + \sum_{j=1}^{n} \ell(J_j) - b_0(X)(M - m)\,, \qquad (1)$$

Results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ratio: 0.508471
ratio: 0.095630
ratio: 0.716759
10000
8000
6000
4000
2000
0
0
1
2
3

Thanks for Your Attention!